Received: *January 21, 2000*, Accepted: *February 22, 2000*

Gauge unification in the dual standard model

Nathan F. Lepora

Department of Applied Mathematics and Theoretical Physics
Cambridge University, England
E-mail: nfl10@damtp.cam.ac.uk

Abstract: We calculate the gauge couplings in the dual standard model. These values are consistent with an associated GeV mass scale, and predict the weak mixing angle to be $\sin^2\theta_w(M_Z) \sim 0.22$.

Keywords: Solitons Monopoles and Instantons, Spontaneous Symmetry Breaking, Duality in Gauge Field Theories, Gauge Symmetry.

JHEP02(2000)036

The standard model fermions have an intricate representation structure under the colour, weak isospin and hypercharge symmetry groups. The five basic multiplets (replicated in three generations) divide into leptons and quarks corresponding to trivial and fundamental representations of the colour symmetry. These leptons and quarks subdivide further corresponding to the trivial or fundamental representations of weak isospin, with this division coinciding with left and right parity eigenstates.

Currently, the *only* explanation for such a structure is the dual standard model of Vachaspati [1]. Here the standard model fermions are associated with monopoles originating from the symmetry breaking of a unified SU(5) gauge theory to the standard model gauge symmetry. The representation structure, and hence interaction, of these monopoles is in exact agreement with the spectrum of fermions in the standard model.

In addition, other properties such as the spin [2], and the number of generations can be consistently included within this framework [3]. Its structure may also be related to confinement within QCD [4, 5].

In this letter we calculate the colour, weak isospin and hypercharge gauge coupling constants of these SU(5) monopoles. Essentially our calculation compares the gauge transformation properties of the monopoles with the associated fermions. This uniquely specifies the colour, weak isospin and hypercharge gauge coupling constants in terms of the unified SU(5) coupling. We find these gauge couplings to be consistent with the experimental values.

We begin by summarising some of the main features of the dual standard model [1, 5]. The model originates with a breaking of SU(5) gauge symmetry

$$\mathrm{SU}(5) \to \mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2)) = [\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)]/\mathbb{Z}_6 \tag{1}$$

and has a monopole spectrum corresponding to the homotopy classes

$$\begin{aligned}\pi_2\left(\frac{\mathrm{SU}(5)}{\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))}\right) &\cong \pi_1(\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))) \\ &= \mathbb{Z}_6 \times \mathbb{Z}\,. \end{aligned} \tag{2}$$

Here $\mathbb{Z}$ defines the degree of the homotopy class, whilst

$$\mathbb{Z}_6 = \mathbb{Z}_3 \times \mathbb{Z}_2 = \{e^{2i\pi/3}, e^{-2i\pi/3}, 1\} \times \{-1, 1\} \tag{3}$$

represents second homotopy classes of same degree.

The monopoles spectrum is built up from bound states of embedded SU(2) $\to$ U(1) fundamental monopoles,

$$\begin{array}{ccc} \mathrm{SU}(5) & \to & \mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2)) \\ \cup & & \cup \\ \mathrm{SU}(2) & \to & \mathrm{U}(1)\,, \end{array} \tag{4}$$

and correspond to the $(e^{2i\pi/3}, -1)$ homotopy class of $\mathbf{Z}_6$. Gardner and Harvey [6] show that these fundamental monopoles combine to form stable bound states for a natural range of model parameters. Labelling the bound states by their asymptotic magnetic fields

$$B^k \sim \frac{\hat{r}^k}{r^2} Q \,, \tag{5}$$

defines an associated magnetic charge

$$Q = \frac{1}{g_u} \left(q_{\mathrm{C}} T_{\mathrm{C}} + q_{\mathrm{I}} T_{\mathrm{I}} + q_{\mathrm{Y}} T_{\mathrm{Y}} \right) , \tag{6}$$

where $T_{\mathrm{C}}, T_{\mathrm{I}}$ and T_{Y} are suitably normalised elements of the Lie algebras $\mathrm{su}(3), \mathrm{su}(2)$ and $\mathrm{u}(1)$. The coefficient $1/g_u$ relates to the unified $\mathrm{SU}(5)$ gauge coupling g_u.

The magnetic charges are determined by associating them with the corresponding homotopy classes in eq. (2). They define a subgroup

$$\mathrm{U}(1)_Q = \exp(\mathbf{R} Q) \subset \mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2)) \,, \tag{7}$$

normalised by

$$\exp(2\pi g_u Q) = 1 \,. \tag{8}$$

This subgroup represents a typical element of the associated $\mathbb{Z}_6$ homotopy class of the monopole. Using generators

$$T_{\mathrm{C}} = i \,\mathrm{diag} \left(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, 0, 0 \right) \tag{9}$$

$$T_{\mathrm{I}} = i \,\mathrm{diag} \left(0, 0, 0, 1, -1 \right) \tag{10}$$

$$T_{\mathrm{Y}} = i \,\mathrm{diag} \left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2} \right) \tag{11}$$

leads to the pattern for the monopole spectrum given in the table.

It should be noted that we have chosen a slightly different normalisation from [1, 5]. This is to agree with the standard particle physics charge normalisations.

	q_{C}	q_{I}	q_{Y}	d_{C}	d_{I}	d_{Y}
$(e^{2i\pi/3}, -1)$	1	1/2	1/3	3	2	1
$(e^{-2i\pi/3}, 1)$	−1	0	2/3	3	0	1
$(1, -1)$	0	1/2	1	0	2	1
$(e^{2i\pi/3}, 1)$	1	0	4/3	3	0	1
$(e^{-2i\pi/3}, -1)$	–	–	–	–	–	–
$(1, 1)$	0	0	2	0	0	1

Degeneracies $d_{\mathrm{C}}, d_{\mathrm{I}}$ and d_{Y} of the monopole embeddings corresponding to the same homotopy class have also been included. These arise from the degeneracy of suitable generators

$$\begin{aligned} T_{\mathrm{C}}^r &= i \,\mathrm{diag} \left(+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, 0, 0 \right) \\ T_{\mathrm{C}}^g &= i \,\mathrm{diag} \left(-\frac{1}{3}, +\frac{2}{3}, -\frac{1}{3}, 0, 0 \right) \\ T_{\mathrm{C}}^b &= i \,\mathrm{diag} \left(-\frac{1}{3}, -\frac{1}{3}, +\frac{2}{3}, 0, 0 \right) \end{aligned} \tag{12}$$

for $X_{\rm C}$ and

$$T_{\rm I}^{\pm} = \pm T_{\rm I}\,, \tag{13}$$

for $T_{\rm I}$. This indicates the monopoles form representations of $\mathrm{SU}(3)_{\rm C}$, $\mathrm{SU}(2)_{\rm I}$ and $\mathrm{U}(1)_{\rm Y}$ with the corresponding dimension. Namely the fundamental representations.

The above arguments strongly imply that the long range interactions of these monopoles is associated with that of a particle with gauge interactions specified by the fundamental representations of the colour, weak isospin and hypercharge symmetry groups. This particle has the corresponding charges $q_{\rm C}$, $q_{\rm I}$ and $q_{\rm Y}$, and its current $J^{\mu}_{\rm mon}$ couples to the gauge fields as

$$\left[g_{\rm C} q_{\rm C} A^{\mu}_{\rm C} + g_{\rm I} q_{\rm I} A^{\mu}_{\rm I} + g_{\rm Y} q_{\rm Y} A^{\mu}_{\rm Y}\right] J^{\mu}_{\rm mon}\,, \tag{14}$$

with $g_{\rm C}$, $g_{\rm I}$ and $g_{\rm Y}$ representing the respective gauge couplings. Such a spectrum of charges and interactions is completely in accord with the spectrum of fermions in the standard model, with the identification:

$$\begin{aligned}
(e^{2i\pi/3}, -1) &\leftrightarrow (u,d)_L \\
(e^{-2i\pi/3}, 1) &\leftrightarrow \bar{d}_L \\
(1,-1) &\leftrightarrow (\bar{\nu}, \bar{e})_R \\
(e^{2i\pi/3}, 1) &\leftrightarrow u_R \\
(1,1) &\leftrightarrow \bar{e}_L\,.
\end{aligned} \tag{15}$$

The corresponding fermionic anti-particles are associated with the anti-monopoles.

On the question of duality, both the residual symmetry group $\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))$ and its dual $\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))^v = \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ have the same derived representation, because their local structure is equivalent. Thus in both cases the action of their associated gauge fields on particle representations are the same.

The main point of this work is to show that as well as predicting the spectrum and properties of fermions in the standard model, the dual standard model also predicts the corresponding colour, weak isospin and hypercharge gauge couplings. We shall determine these from comparing the gauge couplings of the monopole currents to the corresponding expressions for their associated fermions. We shall also give an alternative, but equivalent, argument from the gauge transformation properties of the monopoles.

One may see simply that three different gauge couplings arise in eq. (14) by considering the normalisation of the monopole charge generators in eqs. (9), (10) and (11). These generators $T_{\rm C}$, $T_{\rm I}$ and $T_{\rm Y}$ are normalised to the topology of $\mathrm{S}(\mathrm{U}(3)\times\mathrm{U}(2))$. However the gauge fields of $\mathrm{SU}(5)$ theory are normalised differently. In the minimal coupling the components of the gauge fields are written

$$D^{\mu} = \partial^{\mu} + g_u A^{\mu}_a \hat{T}_a\,, \tag{16}$$

JHEP02(2000)036

with the SU(5)-basis $\{\hat{T}_a\}$ orthonormal with respect to the inner product

$$\mathrm{tr}(\hat{T}_a \hat{T}_b) = -\frac{1}{2}\delta_{ab}\,. \tag{17}$$

The difference between these normalisations will produce overall scales associated with the gauge couplings.

We shall illustrate the importance of normalisation with the coupling of standard model fermions to their gauge fields. A fermion f couples to gauge fields through its current j^μ. In particular we shall consider the neutral current components

$$j^\mu_{\mathrm{C}} = \bar{f}\gamma^\mu X_{\mathrm{C}} f\,, \qquad j^\mu_{\mathrm{I}} = \bar{f}\gamma^\mu X_{\mathrm{I}} f\,, \qquad j^\mu_{\mathrm{Y}} = \bar{f}\gamma^\mu X_{\mathrm{Y}} f\,, \tag{18}$$

where the standard generators are

$$\begin{aligned} X_{\mathrm{C}} &= i\,\mathrm{diag}(1,1,-2) \\ X_{\mathrm{I}} &= i\,\mathrm{diag}(1,-1) \\ X_{\mathrm{Y}} &= i\,\mathrm{diag}(1,1)\,. \end{aligned} \tag{19}$$

It is important to take a standard su(3), su(2) and u(1) normalisation

$$\mathrm{tr}(\hat{X}^2_{\mathrm{C}}) = \mathrm{tr}(\hat{X}^2_{\mathrm{I}}) = \mathrm{tr}(\hat{X}^2_{\mathrm{Y}}) = -\frac{1}{2}\,, \tag{20}$$

which we will explicitly include by considering

$$\hat{X}_{\mathrm{C}} = \frac{1}{\sqrt{12}} X_{\mathrm{C}}\,, \qquad \hat{X}_{\mathrm{I}} = \frac{1}{2} X_{\mathrm{I}}\,, \qquad \hat{X}_{\mathrm{Y}} = \frac{1}{2} X_{\mathrm{Y}}\,. \tag{21}$$

Then a fermion with colour charge q_{C}, weak isospin q_{I}, and weak hypercharge q_{Y} has a gauge-current coupling of the form

$$\frac{1}{\sqrt{12}} g_{\mathrm{C}} q_{\mathrm{C}} A^\mu_{\mathrm{C}} j^\mu_{\mathrm{C}} + \frac{1}{2} g_{\mathrm{I}} q_{\mathrm{I}} A^\mu_{\mathrm{I}} j^\mu_{\mathrm{I}} + \frac{1}{2} g_{\mathrm{Y}} q_{\mathrm{Y}} A^\mu_{\mathrm{Y}} j^\mu_{\mathrm{Y}}\,. \tag{22}$$

Now we shall consider the coupling of the corresponding monopoles to their gauge fields. From the above arguments leading to eq. (14) we may take the monopoles as coupling to S(U(3) $\times$ U(2)) gauge fields through their associated currents. In particular we shall consider three neutral components of the monopole current, J^μ_{C}, J^μ_{I}, and J^μ_{Y}. These are associated with generators

$$\begin{aligned} T_{\mathrm{C}} &= i\,\mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, 0, 0\right) \\ T_{\mathrm{I}} &= i\,\mathrm{diag}(0,0,0,1,-1) \\ T_{\mathrm{Y}} &= i\,\mathrm{diag}\left(1,1,1,-\frac{3}{2},-\frac{3}{2}\right)\,. \end{aligned} \tag{23}$$

We shall take a standard su(5) normalisation

$$\mathrm{tr}(\hat{T}_{\mathrm{C}}^2) = \mathrm{tr}(\hat{T}_{\mathrm{I}}^2) = \mathrm{tr}(\hat{T}_{\mathrm{Y}}^2) = -\frac{1}{2}\,, \tag{24}$$

which will be explicitly included by considering

$$\hat{T}_{\mathrm{C}} = \frac{\sqrt{3}}{2} T_{\mathrm{C}}\,, \qquad \hat{T}_{\mathrm{I}} = \frac{1}{2} T_{\mathrm{I}}\,, \qquad \hat{T}_{\mathrm{Y}} = \frac{1}{\sqrt{15}} T_{\mathrm{Y}}\,. \tag{25}$$

Then a monopole with colour charge q_{C}, weak isospin q_{I}, and weak hypercharge q_{Y} has a gauge-current coupling of the form

$$g_{\mathrm{u}} \left[\frac{\sqrt{3}}{2} q_{\mathrm{C}} A_{\mathrm{C}}^{\mu} J_{\mathrm{C}}^{\mu} + \frac{1}{2} q_{\mathrm{I}} A_{\mathrm{I}}^{\mu} J_{\mathrm{I}}^{\mu} + \frac{1}{\sqrt{15}} q_{\mathrm{Y}} A_{\mathrm{Y}}^{\mu} J_{\mathrm{Y}}^{\mu} \right], \tag{26}$$

where the gauge fields are considered as components of the SU(5) gauge field, with a unified coupling g_u.

By associating these monopoles with standard model fermions we associate each monopole current J^{μ} with a corresponding fermion current j^{μ}. We also associate the corresponding gauge fields. Thus the monopole-gauge coupling of eq. (26) and fermion-gauge couplings of eq. (22) are identified. Comparison of the respective coefficients then gives

$$g_{\mathrm{C}} = 3 g_u\,, \qquad g_{\mathrm{I}} = g_u\,, \qquad g_{\mathrm{Y}} = \frac{2}{\sqrt{15}} g_u\,, \tag{27}$$

which predicts the following ratios:

$$\frac{g_{\mathrm{C}}}{g_{\mathrm{I}}} = 3\,, \qquad \frac{g_{\mathrm{Y}}}{g_{\mathrm{I}}} = \frac{2}{\sqrt{15}}\,. \tag{28}$$

Such values represent a specific prediction of the dual standard model and are completely characteristic of it.

The above relation may also be seen from the explicit transformation properties of the monopoles. Recall that the fundamental monopoles are embedded SU(2) $\to$ U(1) monopoles, described by eq. (4), with magnetic fields B^k corresponding to the embedding. Rigid (or global) monopole gauge transformations that respect $B^k \in \mathrm{su}(3)_{\mathrm{C}} \oplus \mathrm{su}(2)_I \oplus \mathrm{u}(1)_{\mathrm{Y}}$ transform

$$B^k \mapsto \mathrm{Ad}(h) B^k \tag{29}$$

under the adjoint action of $h \in \mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2))$. Correspondingly the su(2) embedding transforms under

$$\mathrm{su}(2) \mapsto \mathrm{Ad}(h)\, \mathrm{su}(2)\,, \tag{30}$$

so that Q transforms appropriately.

Consider a rigid gauge transformation of the embedded monopole, with the generators normalised as in eq. (24)

$$B^k \mapsto \mathrm{Ad}\left[\exp\left(g_u\left(\hat{T}_{\mathrm{C}}\theta_{\mathrm{C}} + \hat{T}_{\mathrm{I}}\theta_{\mathrm{I}} + \hat{T}_{\mathrm{Y}}\theta_{\mathrm{Y}}\right)\right)\right] B^k \,. \tag{31}$$

Those taking $B^k \mapsto B^k$ are thus

$$\theta_{\mathrm{C}} = \frac{2}{\sqrt{3}}\frac{2\pi}{g_u}n_{\mathrm{C}}\,, \qquad \theta_{\mathrm{I}} = 2\frac{2\pi}{g_u}n_{\mathrm{I}}\,, \qquad \theta_{\mathrm{Y}} = \sqrt{15}\frac{2\pi}{g_u}n_{\mathrm{Y}}\,, \tag{32}$$

with each $n \in \mathbb{N}$.

Now we associate the above transformation with an analagous rigid gauge transformations on a fermion f in the fundamental representation of $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{I}} \times \mathrm{U}(1)_{\mathrm{Y}}/\mathbb{Z}_6$

$$f \mapsto e^{g_{\mathrm{Y}}\bar{\theta}_{\mathrm{Y}}\hat{X}_{\mathrm{Y}}} \exp(g_{\mathrm{C}}\bar{\theta}_{\mathrm{C}}\hat{X}_{\mathrm{C}})\, f\, \exp(g_{\mathrm{I}}\bar{\theta}_{\mathrm{I}}\hat{X}_{\mathrm{I}})\,, \tag{33}$$

with g_{C}, g_{I} and g_{Y} the colour, weak isospin and hypercharge gauge couplings. Those rigid gauge transformation that take $f \mapsto f$ are thus

$$\bar{\theta}_{\mathrm{C}} = \sqrt{12}\,\frac{2\pi}{g_{\mathrm{C}}}n_{\mathrm{C}}\,, \qquad \bar{\theta}_{\mathrm{I}} = 2\frac{2\pi}{g_{\mathrm{I}}}n_{\mathrm{I}}\,, \qquad \bar{\theta}_{\mathrm{Y}} = 2\frac{2\pi}{g_{\mathrm{Y}}}n_{\mathrm{Y}}\,, \tag{34}$$

with each $n \in \mathbb{N}$.

Equating monopole and fermion gauge transformation identifies each θ and $\bar{\theta}$ in eqs. (32) and (34). This again gives the ratios found in eq. (27).

These predictions are compared to the running gauge couplings through the following plot. The strong coupling is taken from a three loop calculation normalised to $g_{\mathrm{C}}(M_Z) = 1.213$. The hypercharge and weak isospin are taken from one loop expressions normalised to $g_{\mathrm{I}}(M_Z) = 0.661$ and $g_{\mathrm{Y}}(M_Z) = 0.354$.

We shall make a couple of comments about the running of the gauge couplings in the standard model. Firstly, around $10^{15} - 10^{18}$GeV, when $g_{\mathrm{C}}/g_{\mathrm{I}} \sim 1$ then also $g_{\mathrm{Y}}/g_{\mathrm{I}} \sim \sqrt{3/5}$, as required for grand unification. Secondly, we estimate $\sin\theta_w$ from the W-mass [7], considering the radiative corrections to vary with the running of the fine structure constant α

$$M_W = \frac{A_0}{\sin\theta_w(1-\Delta r)^{1/2}}\,, \qquad A_0 = \left(\frac{\pi\alpha}{\sqrt{2}G_F}\right)^{1/2}\,, \tag{35}$$

with Δr representing the radiative corrections. Its component from the running of α is $\Delta r_0(\mu) = (1 - \alpha/\alpha(\mu))$.

The conclusion from figure 1 is that the dual standard model is associated with a mass scale of around a few GeV. At that scale the running couplings take the values of *both* of our theoretical predictions in eq. (28).

To illustrate the accuracy of the fit in figure 1 we shall calculate a prediction for $\sin^2\theta_w(M_Z)$ using only the running of the strong coupling and eq. (28). Firstly

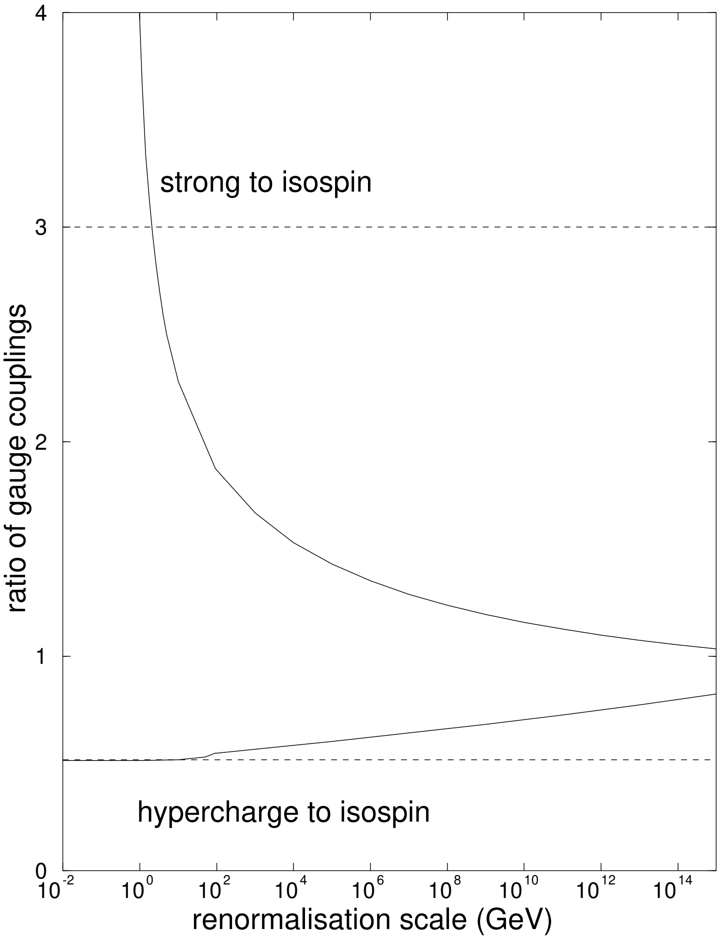

Figure 1: $g_{\rm C}/g_{\rm I}$ and $g_{\rm Y}/g_{\rm I}$ plotted against renormalisation scale μ. Dual standard model predicted values are also included.

observe that $g_{\rm C}/g_{\rm I} = 3$ is satisfied at around a few GeV. Then eq. (28) implies that $\sin^2\theta_w = 4/19$ at the same scale. Using eq. (35), we predict

$$\sin^2\theta_w(M_Z) \sim \frac{\alpha(M_Z)}{\alpha}\sin^2\theta_w(0) \sim 0.22\,. \tag{36}$$

The experimental value is $\sin^2\theta_w(M_Z) = 0.2230 \pm 0.0004$.

The above is the conclusion of this work. We considered the long range interactions of monopoles in the dual standard model to be given via the colour, isospin and hypercharge gauge fields. Then, by an appropriate appreciation of the associated normalisations of the gauge fields, we derived relations between the colour, isospin and hypercharge gauge fields at the scale of monopole unification. These values were found to be consistent with standard model gauge couplings, and the degree of their consistency may be appreciated through the prediction of $\sin^2\theta_w(M_Z)$ in eq. (36).

We think that the above results should be appreciated independently of any interpretation placed on them. It is our aim to present the above mathematics as self consistent, and arising through the geometric structure of the monopoles occuring in Georgi-Glashow SU(5) theory. However, since the agreement is so precise one must speculate somewhat on the fundamental structure that gives rise to this agreement. This is the subject of the rest of this letter, although it should be appreciated that the results we have presented thus far should be considered independently of the following discussion. Indeed, all of the following interpretations may be incorrect.

A first interpretation is coincidence. One may achieve no further implication from such an interpretation.

A second, conventional, interpretation is the relations are arising from some duality between the standard model fermions and the non-perturbative features of Georgi-Glashow SU(5) symmetry breaking. Presumably such duality gives rise to consistency relation in the gauge coupling constants of the standard model. The existence of such a duality is most likely to arise within a string, or some membrane theory, where examples of analogous dualities are known. In this context the relations between the standard model gauge couplings constants that we have derived could be interpreted as a direct low energy implication of the fundamental string or membrane unification picture.

We believe to give weight to such a proposal a specific duality of the fundamental unification would need to be found. Such a question is an interesting proposal, and in our opinion should be investigated further. However, it is beyond the scope of the present paper to discuss this further.

A third, and more unconventional, interpretation is the observed fermions of the standard model really are monopoles. They are formed at monopole unification, where the fundamental gauge symmetries unify. Above this scale there are no fermions, and matter exists solely in the form of fundamental fields that make up the unified gauge theory.

The necessary, and dramatic feature of such an interpretation is that gauge unification occurs at a plasma temperature of around a few GeV. Clearly this feature seems problematic, indeed is completely contradictory to the present viewpoint on unification. However, such plasma temperatures have not yet been reached and, in our opinion, until they have the consequences of such a suggestion should be explored.

It should be noted that in one context unification at a few GeV is desirable. Typical masses of the monopoles are of the unification scale, which is a fairly typical mass scale of the standard model fermions, somewhere between the charmed and bottom quark masses. Thus at least this mass scale is consistent with the fermion masses. In fact, in this context, if monopole unification scale were much higher it would be difficult to reconcile with the observed fermion masses.

In conclusion we have examined the gauge couplings in the dual standard model. This model represents a theoretically well motivated explanation of the spectrum and interaction of the standard model fermions. We have shown that it predicts $g_{\mathrm{C}}/g_{\mathrm{I}} = 3$ and $g_{\mathrm{Y}}/g_{\mathrm{I}} = 2/\sqrt{15}$, values that are consistent with the standard model gauge couplings at a renormalisation scale around a few GeV.

Acknowledgments

I acknowledge King's College, Cambridge for a junior research fellowship and thank B. Allanach, P. Saffin and T. Vachaspati for their valuable comments and advice. I also thank M. Ortiz for discussions when this work was in an early state of development.

References

[1] T. Vachaspati, *An attempt to construct the standard model with monopoles*, *Phys. Rev. Lett.* **76** (1996) 188 [hep-ph/9509271].

[2] T. Vachaspati, *Spin from isospin in purely bosonic theories without magnetic charge*, hep-th/9709149.

[3] H. Liu, G.D. Starkman and T. Vachaspati, *Family replication in the dual standard model*, *Phys. Rev. Lett.* **78** (1997) 1223 [hep-ph/9611220].

[4] A.S. Goldhaber, *Dual confinement of grand unified monopoles?*, *Phys. Rep.* **315** (1999) 83 [hep-th/9905208].

[5] H. Liu and T. Vachaspati, SU*(5) monopoles and the dual standard model*, *Phys. Rev.* **D 56** (1997) 1300 [hep-th/9604138].

[6] C.L. Gardner and J.A. Harvey, *Stable grand unified monopoles with multiple Dirac charge*, *Phys. Rev. Lett.* **52** (1984) 879.

[7] J. Erler and P. Langacker, *Electroweak model and constraints on new physics: in review of particle physics (rpp 1998)*, *Eur. Phys. J.* **C 3** (1998) 90.